Intrinsic Rotation from a Residual Stress at the Boundary of a Cylindrical Laboratory Plasma

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An azimuthally symmetric radially sheared azimuthal flow is driven by a nondiffusive, or residual, turbulent stress localized to a narrow annular region at the boundary of a cylindrical magnetized helicon plasma device. A no-slip condition, imposed by ion-neutral flow damping outside the annular region, combined with a diffusive stress arising from turbulent and collisional viscous damping in the central plasma region, leads to net plasma rotation in the absence of momentum input.

The origins of the so-called intrinsic rotation of tokamak plasmas, where the plasma rotates toroidally without any *apparent* input of toroidal momentum, are of significant interest due to the impact of plasma rotation on core plasma confinement [1] and MHD stability limits [2]. Recent studies on the ALCATOR C-MOD device [3] show that these flows have their origins at the boundary of the tokamak plasma, while work on DIII-D demonstrates that sustaining a tokamak plasma with zero toroidal angular rotation requires a finite countercurrent toroidal momentum input [4]. It has recently been proposed that intrinsic rotation can be caused by the interaction of a turbulent shear stress at the plasma boundary with a no-slip boundary condition at the plasma edge due, for example, to momentum exchange with a stationary neutral gas localized in the boundary region near the wall [5,6]. Measurements in the Joint European Torus (JET) suggest that edge plasma turbulent momentum transport is strong enough to materially affect the edge plasma flows [7]; however to date there have been no direct measurements of nondiffusive, or residual shear stress at the plasma boundary.

The significance of the residual stress can be seen by writing the total stress $S_{r\theta}^{\text{tot}}$ as

$$S_{r\theta}^{\text{tot}} = -\chi_\theta \frac{\partial \langle V_\theta \rangle}{\partial r} + V_r^{\text{eff}} \langle V_\theta \rangle + S_{r\theta}^{\text{res}} \qquad (1)$$

where the terms on the right denote the diffusive stress arising from turbulent momentum diffusion and two nondiffusive stresses: the stress due to a possible radially directed pinch of azimuthal momentum and a residual stress term, respectively [8]. Here $\langle V_\theta \rangle$ denotes the time-averaged azimuthal plasma fluid velocity and the total stress is given by the turbulent Reynolds stress, $S_{r\theta}^{\text{tot}} = \langle \tilde{v}_r \tilde{v}_\theta \rangle$; both quantities are measured. In the absence of collisional viscosity and ion-neutral flow dissipation (the effects of which are discussed later in this Letter), the turbulent azimuthal momentum balance equation for the

plasma fluid is given as

$$\frac{\partial \langle p_\theta \rangle}{\partial t} = -\frac{\partial}{\partial r}(\langle n \rangle S_{r\theta}^{\text{tot}})$$
$$= -\frac{\partial}{\partial r}\left\{ -\chi_\theta \langle n \rangle \frac{\partial \langle V_\theta \rangle}{\partial r} + V_r^{\text{eff}} \langle n \rangle \langle V_\theta \rangle + \langle n \rangle S_{r\theta}^{\text{res}} \right\}$$
$$(2)$$

where $\langle p_\theta \rangle$ is the time-averaged azimuthal momentum and $\langle n \rangle$ is the time-averaged density.

We can now easily see why the residual stress term is crucial to the origin of intrinsic plasma rotation. Consider the idealized case of a dissipation-free plasma with no intrinsic rotation at $t = 0$, i.e., the case where $\frac{\partial \langle V_\theta \rangle}{\partial r} = 0$ *and* $\langle V_\theta \rangle = 0$ everywhere in the plasma. Then we clearly have $\frac{\partial \langle p_\theta \rangle}{\partial t} = -\frac{\partial}{\partial r}\{\langle n \rangle S_{r\theta}^{\text{res}}\}$. Integrating over the plasma radius $\bar{p}_\theta \equiv \int_0^a \langle p_\theta \rangle dr$ gives $\frac{\partial}{\partial t} \bar{p}_\theta = -\langle n \rangle S_{r\theta}^{\text{res}}|_a$ where we have taken $S_{r\theta}^{\text{res}}|_{r=0} = 0$ by symmetry and $r = a$ denotes the plasma boundary. The crucial role for $S_{r\theta}^{\text{res}}|_{r=a}$ is now clear: the only way for the plasma column to acquire a net momentum is for $S_{r\theta}^{\text{res}}|_{r=a}$ to be finite at the boundary.

In earlier work [9,10] we showed evidence that the time-averaged total turbulent Reynolds stress $S_{r\theta}^{\text{tot}} = \langle \tilde{v}_r \tilde{v}_\theta \rangle$ was self-consistent with the observed average radially sheared azimuthal plasma flow and estimated flow damping profiles found in a simple laboratory plasma device, as would be expected for a turbulence-driven zonal flow. The observed plasma rotation is in the electron diamagnetic drift direction without any external momentum input, a similar behavior to the tokamak results summarized above.

In this Letter we report *three* distinct observations that taken together show that those earlier observations can be explained by a residual shear stress localized to the plasma boundary which drives sheared azimuthal flow combined with a diffusive stress that couples this driven edge flow with the central plasma. *First*, when the diffusive turbulent momentum stress term (which always acts to dissipate azimuthal flow) is synthesized from turbulence measure-

ments and subtracted from $S_{r\theta}^{\text{tot}} = \langle \tilde{\nu}_r \tilde{\nu}_\theta \rangle$ (which is measured), we infer a finite residual stress localized to the plasma boundary which drives azimuthal rotation. *Second*, observations of slow oscillations of the sheared azimuthal flow profile demonstrate that changes in the azimuthal flow amplitude originate at the plasma boundary. *Third*, using a newly developed technique [11], we show that the nonlinear transfer of fluctuation kinetic energy into large-scale flows is localized to the same region where we infer the strong boundary residual stress. The latter two observations are independent of the details of the assumed turbulent diffusion term and thus provide independent confirmation of the existence of a residual stress flow drive localized to the boundary region.

Probe measurements and fast imaging show the existence of eddylike structures in the mean gradient region of the plasma [12] which on average persist for a correlation time and then dissipate. Let us therefore assume that the turbulent momentum diffusivity can be expressed in terms of the eddy radial velocity $\tilde{\nu}_r$ and eddy correlation time τ_C via the relation $\chi_\theta = \langle \tilde{\nu}_r^2 \rangle \tau_c$. Furthermore, since the plasma is azimuthally symmetric and the magnetic field is straight and uniform along z direction, we argue that the inward pinch term in the momentum equation vanishes [8]. The residual stress profile is then given as

$$S_{r\theta}^{\text{res}} = \langle \tilde{\nu}_r \tilde{\nu}_\theta \rangle + \langle \tilde{\nu}_r^2 \rangle \tau_c \frac{\partial \langle V_\theta \rangle}{\partial r}$$

where, in principle, all of the terms depend upon position. Direct measurements permit evaluation of the terms on the right-hand side.

A Langmuir probe array has been used to measure the relevant radial profiles in the CSDX device (Fig. 1) in which a 4.5 cm radius $m = 0$ helicon source in a ~ 2.7 m long uniform 0.1 Tesla magnetic field mated to a 10 cm radius 2.7 m long vessel generates a $\sim 10^{13}$ cm^{-3}/3 eV argon plasma with the source operating at 13.56 MHz/1.5 kW with a 4 mTorr gas pressure. Details of the device, plasma conditions and probe arrays, and transition to drift waves and drift turbulence can be found elsewhere [9–14]. The time-averaged plasma density peaks at $r = 0$ and has a strong gradient in the region near $r = 3$ cm [Fig. 1(a)]. As discussed in earlier publications [10] the radial pressure gradient location is defined by the plasma source radius, which is mapped along the field lines to $r \approx 4$ cm for the axial location shown here, and which determines the region of heat input into the system. The time-averaged shear flow profile measured using a time-delay estimation technique [15], $\langle V_\theta(r) \rangle$, has a peak shearing rate located in the region between $r = 3.5$–4 cm [Fig. 1(f)]. As shown elsewhere, this measured velocity is composed of a smaller electron diamagnetic drift component and a larger fluid flow associated with an $m = 0$ azimuthal $E_r \times B_z$ flow which is also in the electron drift direction [16]. Thus here we have taken the lab-frame group velocity found from azimuthally separated probes as a proxy for the fluid flow. The total Reynolds stress,
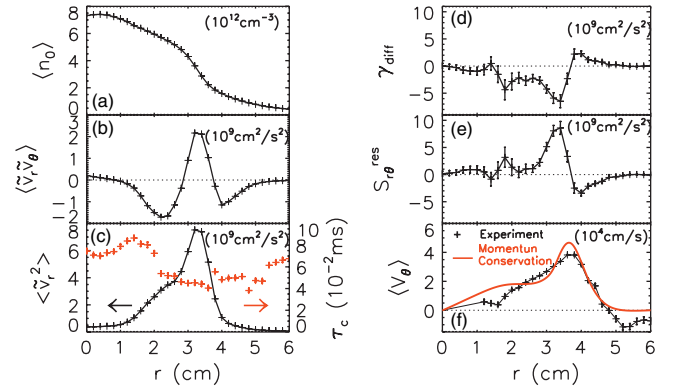

FIG. 1 (color online). Radial profiles of (a) mean density, (b) measured total Reynolds Stress $S_{r\theta}^{\text{tot}}$, (c) mean-squared radial turbulent velocity $\langle \tilde{\nu}_r^2 \rangle$ (black) and autocorrelation time for the $m > 0$ finite azimuthal mode number $\tilde{\nu}_r$ fluctuations with frequency $f > 5$ kHz (red), (d) diffusive momentum flux $\gamma_{\text{diff}} = -\langle \tilde{\nu}_r^2 \rangle \tau_c \frac{\partial \langle V_\theta \rangle}{\partial r}$, (e) residual stress $S_{r\theta}^{\text{res}}$, and (f) mean azimuthal flow $\langle V_\theta(r) \rangle$ from experiments and momentum conservation calculation. Data obtained for 1000 G, 4 mTorr Argon, 1.5 kw source power at location of $z \sim 85$ cm from the source exit plane.

$S_{r\theta}^{\text{tot}} = \langle \tilde{\nu}_r \tilde{\nu}_\theta \rangle$, shown in Fig. 1(b), has a somewhat complex shape with a positive peak at $r \sim 3$ cm and two negative peaks located at $r \sim 2$ cm and $r \sim 4$ cm, as we have reported earlier [9,10].

The measured mean-squared radial turbulent velocity $\langle \tilde{\nu}_r^2 \rangle$ peaks at $r \sim 3.3$ cm [Fig. 1(c) (black)] and has a decorrelation time τ_c (determined from the e-folding width of the envelope of the autocorrelation function computed from the $m > 0$ finite azimuthal mode number $\tilde{\nu}_r$ fluctuations with frequency $f > 5$ kHz) shown in Fig. 1(c) (red). These measurements then allow the diffusive momentum flux, $\gamma_{\text{diff}} = -\langle \tilde{\nu}_r^2 \rangle \tau_c \frac{\partial \langle V_\theta \rangle}{\partial r}$ to be synthesized. This flux, shown in Fig. 1(d), always carries momentum away from the shear layer. Subtracting the diffusive momentum flux from the measured total stress then yields the residual stress $S_{r\theta}^{\text{res}}$ shown in Fig. 1(e). Note that $S_{r\theta}^{\text{res}}$ is negligible for $r < 2.5$ cm, and then peaks at $r \sim 3.3$–3.5 cm with a maximum value that is significantly larger than the error computed from the combination of the total stress and diffusive stress errors. The residual stress changes sign near $r = 4$ cm, and then decays to small values for $r > 5$ cm. Thus we see that the total Reynolds stress can be decomposed into a residual stress localized to a narrow (<1 cm) annular region located at or just inside the plasma edge and a diffusive flux of angular momentum. Of course this conclusion depends upon the assumed form for χ_θ and the assumption that the pinch term is negligible. Thus before we can firmly conclude that such a residual stress indeed exists and influences the plasma flow, other confirming observations are necessary.

Recent work has shown that the time-averaged shear layer behavior shown in Fig. 1(f) can exhibit transient behavior in which the azimuthal velocity increases by a

factor of 2–3 above the average value and then relaxes back to a lower value [17]. These transient bursts of sheared flow have also been shown to be associated with corresponding bursts in the total stress [12]. Parallel fluctuation correlation lengths have been shown to be several meters [18,19], allowing axial imaging of the quasi-2D fluctuations in the (r, θ) plane. Using a 25 cm diameter $f/10$ telescope viewing parallel to the magnetic field we have measured the visible light intensity fluctuation velocity field using time-delay estimation techniques applied to high speed (100 000 frames/second) imaging [15]. The resulting two-dimensional flow field can be averaged over azimuthal position to then yield a measurement of $V_\theta(r, t)$. The results (Fig. 2) show that the transient increase in $V_\theta(r, t)$, is most pronounced and occurs first in the region $3 < r < 4$ cm, suggesting that the burst of shear flow originates there. The flow profile evolution in the region $r < 3$ cm (Fig. 2 inset) then suggests an inward penetration of azimuthal velocity into this region from the edge region. These dynamics are consistent with the effect of a negative divergence (i.e., a convergence) of the residual stress that drives the edge flow [see Eq. (2)] which then couples to the central plasma via the inward radial diffusion of azimuthal flow.

A third independent check that the residual stress is localized to the boundary can be obtained using a recently developed technique which measures the nonlinear kinetic energy transfer between different frequency ranges [11]. The $m \geq 1$ drift fluctuations occur in the frequency range $f > 3$–5 kHz while the slowly varying $m = 0$ shear flow occurs in the frequency range $f < 1$–2 kHz. Thus we can then directly measure the nonlinear transfer of kinetic energy from velocity fluctuations $\tilde{\mathbf{v}}$ with $f_1 > 3$–5 kHz into a large-scale velocity V_θ in the frequency range $f < 1$ kHz by measuring the cross bispectrum with a dual 3×3 probe array (3 mm $\times$ 5 mm in the $r - \theta$ plane) and then

summing over the appropriate frequency ranges $T_{V_\theta} = \sum_{\substack{f < 1\,\text{kHz} \\ f_1 > 5\,\text{kHz}}} \langle V^*(f)\tilde{\mathbf{v}}(f - f_1) \cdot \nabla \tilde{\mathbf{v}}(f_1) \rangle$. Here $\langle \cdots \rangle$ denotes an ensemble average over a sufficiently large (~ 1000) number of independent ensembles and $\mathbf{v} = \frac{-\nabla \phi \times B_0}{B_0^2}$ is the $E \times B$ drift due to the gradient in electrostatic potential. The radial variation of T_{V_θ} can then be measured by moving the array across the plasma. The result (Fig. 3) indicates that $T_{V_\theta} > 0$ at $2.8 < r < 4$ cm consistent with a low frequency shear flow driven by the higher frequency fluctuations; at other radii $T_{V_\theta} \lesssim 0$, suggesting a transfer of kinetic energy from the large-scale flow into the smaller scaled velocity fluctuations as would be expected for turbulent viscous momentum dissipation. The magnitude of the $T_{V_\theta} < 0$ data points are ~ 10 smaller than the peak $T_{V_\theta} > 0$ values consistent with a rate of nonlinear flow drive that is stronger than the central plasma turbulent dissipation rate. We note that $T_{V_\theta} > 0$ is also consistent with the notion of a negative turbulent viscosity phenomenon, driven, in this case, by the residual stress at the plasma boundary.

Examining $S_{r\theta}^{\text{res}}$ [Fig. 1(e)] and Eq. (2), in the absence of flow dissipation we would then expect a positively directed flow (electron drift direction in our sign convention) in the $3.4 < r < 4.2$ cm region where $\nabla_r S_{r\theta}^{\text{res}} < 0$, and a negatively directed flow for $r < 3.4$ cm and for $r > 4$ cm where $\nabla_r S_{r\theta}^{\text{res}} > 0$. The actual flow profile [data points shown in Fig. 1(f)] shows positive average azimuthal rotation for $r < 4.6$ cm and negative rotation for $r > 4.6$ cm; the strongest shearing occurs at $r \sim 3.5$–3.7 cm, and the plasma has net rotation dominated by core plasma rotation in the electron diamagnetic direction—an expectation that is at odds with the results shown in Fig. 1(f).

However, dissipation cannot be neglected. We estimate that the turbulent momentum diffusion coefficient $\chi_\theta \approx 2$–4×10^5 cm^2/sec is slightly larger than the peak estimated collisional ion-ion viscosity ($\mu_{ii}|_{\text{max}} \sim 10^5$ cm^2/sec) [9,10], resulting in a radial turbulent momentum diffusion time scale $\tau_\theta^{\text{diff}} \approx \frac{a^2}{\mu_{ii} + \chi_\theta} \approx 50$–100 μsec.
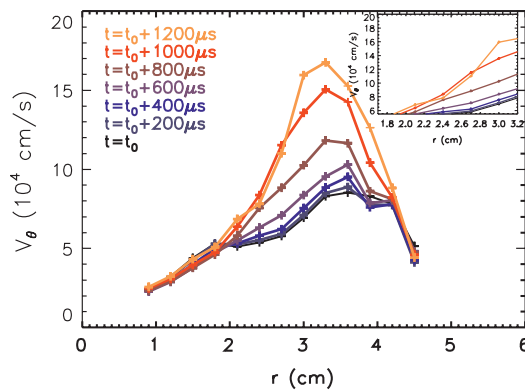

FIG. 2 (color online). Radial profiles of the time-varying azimuthal velocity fields for a period of time from t_0 to $t_0 + 1200$ μs. The insert is a zoom in of the profile from $r = 1.8$ cm to $r = 3.2$ cm. Data obtained for 1000 G, 3 mTorr Argon 1.5 kW conditions, which give slightly higher azimuthal flows that the conditions found in Fig. 1 due to the reduced ion-neutral flow damping.
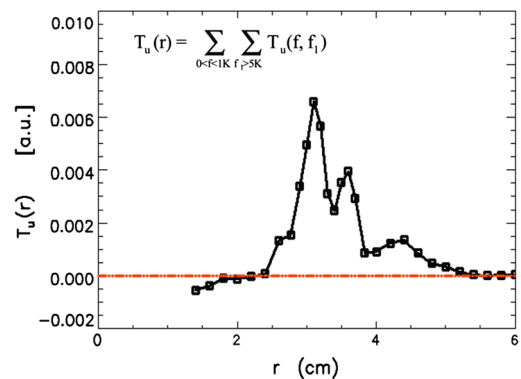

FIG. 3 (color online). Radial profile of the nonlinear kinetic energy transfer from the drift wave fluctuations ($f > 5$ kHz) into the slowly varying $m = 0$ flow located at $f < 1$ kHz.

Estimates [9,10] and measurements [20] show that strong on axis neutral density depletion (by a factor $\sim T_{\text{gas}}(r = 0)/T_{\text{gas}}(r = r_{\text{wall}}) \sim 35$) occurs in these discharge conditions yielding a central plasma ion-neutral damping time scale $\tau_{io} = \frac{1}{n_{\text{gas}}\sigma_{io}^{\text{tot}} V_\theta} \approx 200 \, \mu$ sec as shown previously [9,10]. The combined effects of turbulent and collisional momentum diffusion and, more weakly, ion-neutral damping, then dominate the residual stress for $r < 3$ cm, and act to couple the residual stress-driven sheared edge flow in the $3 < r < 4$ cm region with the central plasma on the $\tau_\theta^{\text{diff}}$ time scale consistent with the results in Fig. 2.

Outside the plasma boundary the neutrals are nearly in thermal equilibrium with the wall located at $r = 10$ cm (see, e.g., [21]) and must therefore have a density of $\sim 10^{14}$ cm^{-3} from the measured gas pressure. Ion-neutral momentum exchange then gives a mean free path $\lambda_{io}^{\text{mfp}}|_{r>a} = \frac{1}{n_{\text{gas}}\sigma_{io}^{\text{tot}}}|_{r>a} \approx 0.1$ cm $\ll 2\pi a$ in this region. Thus in the outer regions the ion-neutral flow damping imposes an effective no-slip boundary condition $\bar{V}_\theta|_{r>r'} = 0$ at a radius $r' > a$ lying in the outer plasma region where the ion density is small. Examining Fig. 1(f), we estimate that this location must lie in the region between $r = 5$–6 cm. The precise flow distribution of course depends in detail upon the neutral gas density and ion-ion dissipation profiles as we have discussed in detail elsewhere [9,10,17,22]. The solid line in Fig. 1(f) shows the rotation profile that is then expected to develop from the residual stress, diffusive stress and estimated flow dissipation profiles as we have discussed earlier [9,10,22]. Detailed measurements of the ion and neutral profiles, which can be used to provide the collisional dissipation profiles, are planned and will be reported in future work.

Taken together, these three independent experimental approaches point to a picture in which a radially sheared $m = 0$ azimuthal flow in the electron diamagnetic direction is driven by a turbulent residual stress located in a narrow annular layer at or just inside the plasma boundary defined by the source heat input location. The resulting azimuthal flow then diffuses inward towards the plasma center and outward towards the wall due to the combined action of turbulent diffusion and ion-ion collisional viscosity. Strong ion-neutral flow damping in the weakly ionized ($\frac{n_{\text{ion}}}{n_{\text{gas}}} \approx 0.001$–0.01) region located outside of the residual stress layer imposes a net no-slip boundary condition in this region. The combination results in a net intrinsic azimuthal rotation of the plasma column in the absence of external azimuthal momentum input. Similar effects have been proposed to operate in the tokamak plasma edge region [5,6].

The question then arises: What physical process generates the residual stress at the plasma boundary? Several possible hypotheses need to be investigated. For example, recent work in this device using fast camera imaging indicates that bursts of plasma ("blobs") are born near the $r = 4$ cm region [23,24] and then propagate into the $r > 4$–5 cm region [24]. Thus the generation location of the blobs coincides with the location of the residual stress, suggesting a possible link between blob generation and residual stress at the plasma boundary. Similar blob ejection occurs in tokamak experiments [25] and in toroidal laboratory devices [26,27] suggesting that turbulent processes may play a role in edge plasma flows in tokamaks. Recent numerical simulations of the tokamak plasma edge region suggest that the central plasma region could receive a recoil impulse if the blobs are ejected with a preferred azimuthal direction [28]. Such an effect would appear as an effective shear stress applied at the plasma boundary. Alternatively, radial wave propagation arising from symmetric breaking at the plasma boundary has also been proposed to generate a residual stress at the boundary [29] Demonstrating which, if any, of these mechanisms leads to the formation of a residual stress at the boundary region requires additional laboratory plasma work. Similar experiments in tokamak devices would help determine whether similar physics are important for intrinsic rotation in toroidally confined plasmas.

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